Acoustic Particle Velocity Applications

In-situ Surface Impedance and Reflection Coefficient Method

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Introduction to particle velocity

Practical applications

In-Situ absorption estimation based on Equivalent Source Method

Results and discussion
INTRODUCTION
Microflown sensor technology
THE MICROFLOWN SENSOR

Measuring particle velocity

1. Two platinum wires heated up to appr. 200 °C
2. As the air flows through the upstream wire, air temperature increases and the wire cools down.
3. Next, the heated air flows through the downstream wire, again the temperature of the wire drops. However, the decrease is lower than it was with the first wire.
4. The different temperatures of the wires cause different electronic resistances. Finally, the resulting voltage difference over the two wires is measured.
PRESSURE vs PARTICLE VELOCITY

Fundamental physical differences between the two quantities

\[
\begin{align*}
\{ p(r, k) &= \frac{\Delta}{r} e^{-jkr} \\
u_r(r, k) &= -\frac{1}{j\omega} \frac{\partial p(r,k)}{\partial r} = \frac{p(r,k)}{\rho c} (1 + \frac{1}{jkr})
\end{align*}
\]

Near field effect

\[
BC \begin{cases}
\rho^\text{in} + \rho^\text{out} = 0 \\
u_n^\text{in} - u_n^\text{out} = 0
\end{cases}
\]

Figure of 8

\[u_n = \vec{u}.\vec{n} = |\vec{u}| |\vec{n}| \cos(\theta)\]

High surface velocity and low surface pressure

Low surface velocity and high surface pressure

Automatically reduces the energy received by 1/3
3D ACOUSTIC VECTOR SENSOR

Pressure and particle velocity in the X, Y and Z axis

- Acoustic vector sensors (AVS) can be created by using multiple orthogonal particle velocity sensors
- Localization resolution and accuracy is preserved across the frequency spectrum.
- Broad-banded response 20 Hz - 20+kHz
- Sound intensity can be obtained by combinations of all sensor elements

\[
\vec{I} = \frac{1}{2} P \vec{U}^* \\
Z_n = \frac{p}{u_n}
\]
Practical applications
Microflown sensor technology
Sensor Applications

Examples of customer applications

2D Sound Visualization

In-situ absorption

3D Sound Visualization

Sound power

Audio Design

Transmission Loss
SOUND FIELD VISUALIZATION

Analyze results in vector view, scalar view or create as many 2D sound field slices

- Vector field
- Sound pressure
- Sound field slices
AUTOMOTIVE // AUDIO SYSTEM

Sound visualization around the driver’s seat
End of Line // ML Fault Detection

Perform End-of-Line noise tests for objective evaluation, eliminating the variability of the subjective human perception.

Measuring in the particle velocity in the near field allows vibro-acoustic characterization in a noisy environment.
MEASUREMENT METHODOLOGY: ORDER TRANSFORM

Velocity Synchronous Discrete Fourier Transform (VSDFT)

\[ X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} \, dt \]

Domain

\[ X(\Omega) = \int x(t)\omega(t)e^{-j\Omega\phi(t)} \, dt \]

Complex Exponential with RPM related variation

\[ \text{VSDFT}[k] = \frac{\Delta t}{\Theta} \sum_{n=0}^{N-1} x[n\Delta t]\omega[n\Delta t]e^{-j\Omega[k\Delta \Omega]\phi[n\Delta t]} \, dt \]

Fourier Transform

Order Transform

Discrete VSDFT

Time frequency spectrogram → RPM signal → Order based transform → Order spectrum

RPM against frequency spectrogram
MODEL LEARNING: GAUSSIAN MIXTURE MODELS

- Model the distribution of the samples with the objective to will be able to distinguish GOOD from BAD samples.
- Not many samples available 20/20.
- Neural networks wasn’t applicable due sample limitations

\[
p(X \mid \lambda_m) = \sum_{j=1}^{G} \omega_j p_j(X)
\]

\[
p_j(X) = \frac{1}{\sqrt{(2\pi)^N \det(\Sigma_j)}} e^{-\frac{1}{2} [(X-\mu_j)' \Sigma_j^{-1} (X-\mu_j)]}
\]
OUTDOORS LOCALIZATION

Example of application cases
Outdoors localization
Real time source localization

✓ Low frequency noise causes anxiety and insomnia
  • Complaints of about a tonal noise.

✓ Deployment of a network of AMMS:
  • Geolocalization of the problem
  • Temporal and spectral analysis

✓ Example: Cooling system of a factory in Veendam (Netherlands).
  • Tone located at 30 Hz.
  • The system is being replaced.
Outdoors: Wind Turbine

Moving sources beamforming

Velocity potential can be calculated by convolving the excitation signals with the time-varying propagation functions

\[
\Psi_i(x, t) = Q_i(x_0, t) * G(x, x_0, t) = \frac{q_i(t - T)}{4\pi\rho(\|r_i(t)\| - v(t - T) \cdot r_i(t)/c)}
\]

Sound pressure and particle velocity can be directly computed using time and space differentiation

\[
p(x, t) = -\rho \sum_{i=1}^{N} \frac{\partial \Psi_i(x, t)}{\partial t} + n(t)
\]

\[
u(x, t) = \sum_{i=1}^{N} \nabla \Psi_i(x, t) + n(t)
\]
Wind Turbine: Beamforming results

Comparison of microphone array and AVS array: spacing 7 times over Nyquist limit
IN-SITU ABSORPTION
ESM based method
PU in situ method

Motivation

- Extend current in-situ method for impedance estimation to an array of sensors.

\[
\begin{align*}
Q & \quad \theta_1 & & r_1 \\
Q' & \quad \theta_2 & & r_2 \\
r_3 & & & h
\end{align*}
\]
Equivalent Source Method

Array of single later of p-u sensors. Problem definition

Single layer $p-u_{z}$

- sound pressure
- particle velocity

Green functions pressure and particle velocity

- $G(r, r_i) = e^{-jk|r-r_i|}$
- $G^u(r, r_i) = \frac{\partial}{\partial z} G(r, r_i)$

1. Sound field and sources strength relationship

   \[
   \begin{bmatrix}
   p_{h_1} \\
   u_{h_1}
   \end{bmatrix} =
   \begin{bmatrix}
   j\omega \rho G_{q_1 h_1} & j\omega \rho G_{q_2 h_1} \\
   -G^u_{q_1 h_1} & -G^u_{q_2 h_1}
   \end{bmatrix}
   \begin{bmatrix}
   q_1 \\
   q_2
   \end{bmatrix}
   \]
Equivalent Source Method

Equivalent sources strength estimation. Solving inverse problem.

Single layer $p-u_z$

- sound pressure
- particle velocity

\begin{align*}
    \begin{bmatrix}
        p_{h_1} \\
        u_{h_1}
    \end{bmatrix}
    &=
    \begin{bmatrix}
        j\omega \rho G_{q_1 h_1} & j\omega \rho G_{q_2 h_1} \\
        -G_{q_1 h_1}^u & -G_{q_2 h_1}^u
    \end{bmatrix}
    \begin{bmatrix}
        q_1 \\
        q_2
    \end{bmatrix}
    \\
    &= \left(WG\right)^+ Wb
\end{align*}

(1) 2. Solving inverse problem for $q$ (ill-posed)

Where the regularized pseudo-inverse is

\begin{align*}
    \left(WG\right)^+ &= \left([WG^H]^2[WG + \lambda I]\right)^{-1}[WG^H]
    \\
    \text{And the weighting matrix}
    \\
    W &= \left( \begin{bmatrix} ||p_h|| & 0 \\ 0 & ||u_h|| \end{bmatrix} \right)^{-1}
\end{align*}
Equivalent Source Method

Surface impedance and reflection coefficient reconstruction

Single layer $p-u_z$

- sound pressure
- particle velocity

3a. **Sound field reconstructed at the surface** from estimated $q$

- $p_{s0} = j\omega \rho (G_{q1s0} q_1 + G_{q2s0} q_2)$,
- $u_{s0} = -(G^u_{q1s0} q_1 + G^u_{q2s0} q_2)$

3b. **Surface impedance $Z_s$ and reflection coefficient $R$ is computed**

- $Z_{s0} = \frac{1}{N} \sum_{n=1}^{N} \frac{p_{s0}^{(n)}}{u_{s0}^{(n)}}$
- $R_{s0}(\theta) = \frac{Z_{s0} \cos \theta - Z_0}{Z_{s0} \cos \theta + Z_0}$
Equivalent Source Method

Comparison: Single layer – Double layer configuration

- Valid for locally reactive samples only: The impedance doesn’t change with the angle of incidence)
- Works for different types of sources: monopole / dipole
- Doesn’t depend on wave model assumptions like plane wave

Single layer $p-u_z$
- sound pressure
- particle velocity

Double layer $p-p$
- sound pressure

- sound pressure
Equivalent Source Method

Double array of pressure transducers. Problem definition

Double layer $p-p$

- sound pressure

Green functions pressure

1. Sound field and sources strength relationship

\[
\begin{align*}
\mathbf{p}_{h_1} &= j\omega \rho \begin{bmatrix} G_{q_1 h_1} & G_{q_2 h_1} \end{bmatrix} \mathbf{q}_1 \\
\mathbf{p}_{h_2} &= j\omega \rho \begin{bmatrix} G_{q_1 h_2} & G_{q_2 h_2} \end{bmatrix} \mathbf{q}_2
\end{align*}
\]
Acoustic field and impedance model

Pressure and velocity field model above an impedance plane (Di & Gilbert)

- \[ p(\mathbf{r}) = \frac{j\omega \rho Q}{4\pi} \left( \frac{e^{-jk|r-r_1|}}{|r-r_1|} + \frac{e^{-jk|r-r_2|}}{|r-r_2|} - 2k\beta \int_0^\infty e^{k\beta q} e^{-jk\sqrt{d_1^2+(r_{1z}+r_{2z}-j q)^2}} \frac{dq}{d_1^2+(r_{1z}+r_{2z}-j q)^2} \right) \]

- \[ u_z(\mathbf{r}) = -\frac{1}{j\omega \rho} \frac{\partial}{\partial z} p(\mathbf{r}) \]

Porous media model (Delany and Bazley)

- \[ Z_s(f) = Z_0 \left[ 1 + 9.08 \left( \frac{10^3 f}{q} \right)^{-0.75} - j11.9 \left( \frac{10^3 f}{q} \right)^{-0.73} \right] \]

Relative error in dB

- \[ E\{\gamma_{est}\} = 20 \log_{10} \left( \frac{||\gamma_{est}-\gamma_{ref}||_2}{||\gamma_{ref}||_2} \right) \]

Sketch of the geometric parameters
Results and discussion
Results and Discussion
Surface Impedance PU vs PP (SNR 30 dB)

Real part  Surface Impedance

\[ Z_{s0} = \frac{1}{N} \sum_{n=1}^{N} \frac{p_{s0}^{(n)}}{u_{s0}^{(n)}} \]

Relative error

< 10 %  < 10 %  < 10 %  < 10 %
Results and Discussion

Reflection coefficient PU vs PP (SNR 30 dB)

Real part Reflection Coefficient

Imaginary part Reflection Coefficient

\[ R_{s_0}(\theta) = \frac{Z_{s_0} \cos \theta - Z_0}{Z_{s_0} \cos \theta + Z_0}, \]

Relative error < 10 %
Results and Discussion

Relative error behavior Frequency vs SNR

Single Layer P-U method

• **P-U method**: At 400 Hz, < 10 % relative error (-20 dB), -> **SNR Needed**: 15 dB

Dual Layer P-P method

• **P-P method**: At 400 Hz, < 10 % relative error (-20 dB), -> **SNR Needed**: 35 dB
Conclusions

• Complex surface impedance and reflection coefficient have been calculated using ESM in two configurations: single-layer of $p-u$ probes and a double layer of microphones.

• The performance of ESM methods across the frequency for different SNR levels were studied.

• Single layer $p-u$ ESM method has significantly better performance, in special in the low frequency range, compared with the double layer of microphones ESM method.

• In addition, the single layer $p-u$ is also more robust against noise, achieving accurate results with relatively low levels of SNR.
Thank you for your attention

Contact us for further information or visit our website

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